

Effect of Loading Level of Hybrid Reinforced Concrete Deep Beams under Repeated Loading

Sawsan Akram Hassan

Civil Engineering Department, Al-
Mustansiriayah University, Baghdad / Iraq.

Ghsoon Ali Faroun

Civil Engineering Department, Al-
Mustansiriayah University, Baghdad / Iraq.

E-mail of the corresponding author:

dcr.sawsanakram@uomustansiriyah.edu.iq

ghsoonali.90@yahoo.com

**تأثير مستوى التحميل المدعم بالحزم
الخرسانية المسلحة الهجينة تحت التحميل المتكرر**

غصون علي فرعون

قسم الهندسة المدنية ، الجامعة
المستنصرية ، بغداد \ العراق.

سوسن أكرم حسن

قسم الهندسة المدنية ، الجامعة
المستنصرية ، بغداد \ العراق.



Abstract

This research is devoted to investigate the experimental and theoretical behavior of hybrid deep beams under monotonic and repeated two points loading. The dimensions and the flexural reinforcement of the deep beams were kept constant. In this work, the idea of hybrid beam is completely different. Two types of concrete were used but not in cross section. The first type which is fibrous concrete (FC) was used in casting beam sides (shear spans), while the second type was conventional concrete (CC) which was used in the middle portion of beam (between two shear spans). This was done to strengthen the deep beam sides (shear spans) against cracking due to shear (diagonal strut) failure. The experimental program includes casting and testing of 4 deep beams, one of them is tested as a control beam under monotonic loading and the others were tested under repeated loading at level 90%,70% and 50% of the ultimate load of it control beam. The effect of loading type is studied for constant steel fiber (SF) ratio and amount of web reinforcement. The ultimate load decreases when beams subjected to repeated loading of different levels. Comparison of experimental results was made with corresponding predicted values using the Strut and Tie procedure presented in Appendix A of ACI 318M-11Code and with other procedures mentioned in the literature. It was found that the Strut and Tie procedure presented in Appendix A of ACI 318M-11Code give conservative results as compared with the experimental tested results.

Keywords: Deep beam, Hybrid, Repeated loading, Strengthening, Shear spans.

المستخلص

خصص هذا البحث لدراسة السلوك التجريبي والنظري للحزم العميقة الهجينة في ظل التحميل الرتيب والنقطي المتكرر و تم الحفاظ على أبعاد وتقوية الانحناء للحزم العميقة ثابتة في هذا العمل ، أن فكرة الحزم الهجينة مختلفة تمامًا. استخدم نوعين من الخرسانة ولكن ليس في المقطع العرضي. حيث استخدام النوع الأول وهو الخرسانة الليفية (FC) في جوانب عارضة الصب (جسور القص) ، بينما كان النوع الثاني من الخرسانة التقليدية (CC) الذي تم استخدامه في الجزء الأوسط من العارضة (بين فترتي القص). تم عمل ذلك لتقوية جوانب الحزمة العميقة (امتدادات القص) ضد التشقق بسبب فشل القص (الدعامة القطرية). تضمن البرنامج التجريبي صب و اختبار 4 عوارض عميقة ، أختير إحداها كحزمة تحكم تحت تحميل رتيب والآخر تحت التحميل المتكرر عند المستويات 90% و 70% و 50% من الحمل النهائي لحزمة التحكم . تمت دراسة تأثير نوع التحميل لنسبة الألياف الفولاذية الثابتة (SF) وكمية تقوية الويب. و لوحظ انخفاض الحمل النهائي عندما تعرضت الحزم للتحميل المتكرر لمستويات مختلفة. فضلا عن ذلك اجريت مقارنة بين النتائج التجريبية و القيم المتوقعة المقابلة باستخدام إجراء Strut and Tie المقدم في الملحق A من ACI 318M-11Code ومع الإجراءات الأخرى المذكورة في الأدبيات. وجد أن إجراء Strut and Tie الوارد في الملحق A من ACI 318M-11Code يعطي نتائج متحفظة مقارنة بنتائج الاختبار التجريبية.

الكلمات المفتاحية: الحزم العميقة ، هجين ، تحميل متكرر ، تقوية ، قص مسافات.



1- Introduction

Deep beams are structural members differ from slender beams in their geometrical proportions and nowadays are widely used in many structural applications such as transfer girders in multistory buildings, pile supported foundation, foundation walls, shear walls and bridges. Deep beam is defined as those members in which the ratio of effective span (L_n) to depth (h) is less than or equal to four ($L_n/h \leq 4$); or shear span (a) to depth (h) ratio less than or equal to two ($a/h \leq 2$) (ACI,2011: 503) . Reinforced concrete deep beams are typically used as transfer members in high-rise structures due to their high resistance capacity. Because the stress distribution in the section of the deep beam is nonlinear, the linear elastic theory for the general beam analysis cannot be applied. Therefore, ACI 318M-11Code requires that deep beams be designed via non-linear analysis or by Strut and Tie Models (STM) (Noh et al). Hybrid concrete beams are characterized using different types of concrete specific layers for the purpose of increasing the resistance and improve performance.

2- Materials and Methods

The experimental program consists of testing four simply supported deep beams under two point loads to investigate the behavior of reinforced concrete deep beams under repeated loading. All beams have the same dimensions and flexural reinforcement. They had an overall length of 1500 mm, a width of 150 mm and a height of 350 mm. The amount of flexural reinforcement for all the tested beams was ρ and ρ (where ρ is the flexural reinforcement ratio). The clear span between supports was 1230 mm which

results in a ratio of clear span to overall depth of 3.5. The variable include type of load. Also, bearing plates under each load and above each support were designed to avoid any local crushing in concrete. Table (1) show details of the four tested reinforced concrete deep beams. The main parameter investigated and details of the web reinforcement are also shown. Details of dimension and reinforcement for all beam specimens are shown in Figure (1).

Table 1. Beam Specimens Details.

Beam No.	a/h	Type of Beams	Type of Load	SF	Vertical Web. Reinforced	Horizontal Web. Reinforced	ρ_w
B1	1.14	Hybrid	Monotonic	1%	Φ 4 mm @ 80 mm	Φ 4 mm @80 mm	0.003
B2	1.14	Hybrid	Repeated (90% of B5 Ultimate Load)	1%	Φ 4 mm @ 80 mm c/c	Φ 4 mm @80 mm c/c	0.003
B3	1.14	Hybrid	Repeated (70% of B5 Ultimate Load)	1%	Φ 4 mm @ 80 mm c/c	Φ 4 mm @80 mm c/c	0.003
B4	1.14	Hybrid	Repeated (50% of B5 Ultimate Load)	1%	Φ 4 mm @ 80 mm c/c	Φ 4 mm @80 mm c/c	0.003

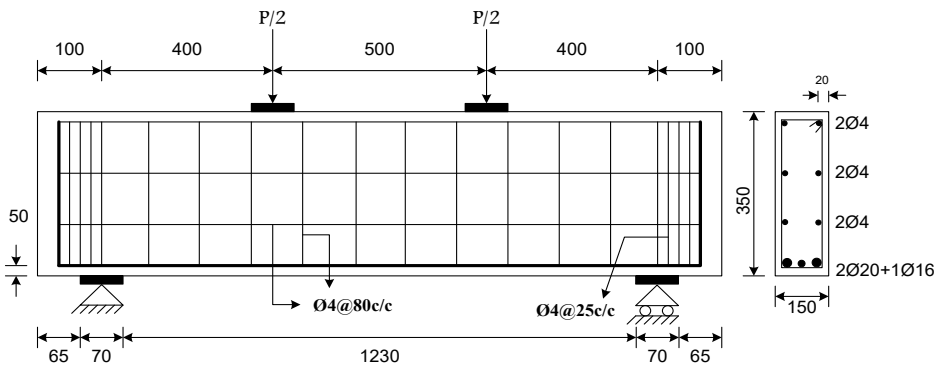


Figure 1. Details of Beams (All Dimensions are in mm).



2.1 Materials

Properties and description of used materials are reported and presented in Table (2) and the concrete mix proportions are reported and presented in Table (3).

Table 2. Properties of Construction Materials.

Material	Descriptions
Cement	Ordinary Portland Cement (Type I)
Sand	Natural sand from Al-Ukhaider region with maximum size of (4.75mm)
Gravel	Crushed gravel of maximum size (19 mm)
Steel Fiber	Hooked ends mild steel fibers are used in construction of fibrous concrete with volumetric ratio (vf) of 0%, 1% and 2%.
Reinforcing Bars	(φ 20mm) deformed steel bar, having (630MPa) yield strength (fy) (φ 16mm) deformed steel bar, having (780MPa) yield strength (fy) (φ 4mm) plane steel bar, having (540MPa) yield strength (fy)
Water	Clean tap water

Table 3. Proportions of Concrete Mix.

Compressive Strength (MPa)	Cement (kg/m ³)	Sand (kg/m ³)	Gravel (kg/m ³)	Water Cement ratio w/c	Steel Fiber (%)	Steel Fiber (kg/m ³)
30	400	728	1092	0.5	0	-
					1	78

2.2 Description of Reinforcement Bars

The horizontal length of all longitudinal reinforcement was 1460 mm and a vertical length of 250mm to make a 90° standard hook to provide sufficient anchorage as shown in Plate (1).

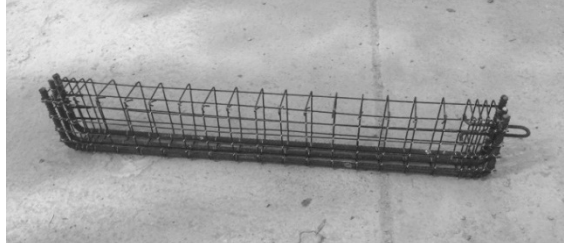


Plate 1. Steel Reinforcement Cage Used for Deep Beams.

2.3 Molds and Casting

Two steel molds were designed and fabricated for casting two hybrid deep beams for each batch. The inside dimensions for each mold were 1500mm ×150mm ×350mm (length × width × depth). The molds were fabricated to cast the beams vertically due to the difficulty of casting layers in horizontal state. The front cover of the mold face of dimensions (1500mm×350mm) consists of three Plates. The lower plate was fixed to cast the first layer while the two other plates above were movable (doors) to cast the two other layers. Each door was closed before casting the layer of beam backwards it. The molds were placed in vertical position and all reinforcing bars were previously prepared. The reinforcement cage then put in position. Details of molds and casting procedure are shown in Plate (2).



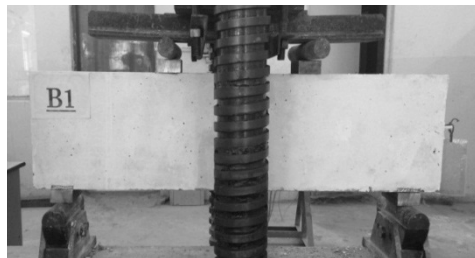
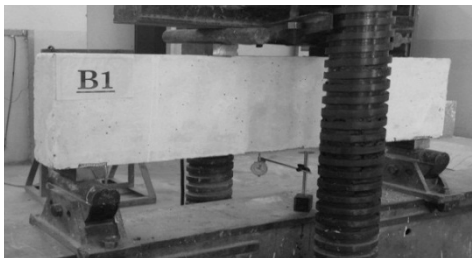
Plate 2. Details of Molds and Casting Procedure.

2.4 Test Procedure

All beam specimens and control specimens have been removed from curing at the age of 28 days. Before the testing day, the beam specimens have been cleaned and paint with white paint in order to clarify the crack propagation. Each beam specimen has been labeled and the locations of support points, loading points and the dial gauge position were marked on the surface of beam. The beam specimens have been placed on the machine with a clear span (1230mm), as shown in Plates (3). The marked loading points have been covered by (150×70×40) mm steel plates to avoid stress concentrations on the upper face of the beams during loading. All beam specimens have been tested under two points load. The dial gauge was mounted in their marked position to touch the bottom of center of the beams was fixed in the correct location. All beam specimens have been loaded to failure in one cycle for monotonic test and 5 cycles in repeated loading test. The beam specimens have been loaded in increments of (10kN), the



rate of load increment was about (1.5kN/sec). The positions and extents of consequent cracks for each cycle were marked on the surface of the beam. As failure occurred, when the beam failed abruptly at simultaneity with the load indicator stopped in recording or return back and the deflection increased very fast. The failure load has been recorded, and the hydraulic load has been removed.



a. Side View for Beam (B1) under Testing.

b. Front View for Beam (B1) under Testing

Plate 3. Test Procedure for Beam Specimen.

3. Results and Discussion

In monotonic loading, during the applied load and at the low load level, all the tested beams behaved in an elastic manner and the deflection at mid span were small proportion to the applied loads. When the load was increased, first crack was occur, then number of cracks were observed at the region of the pure bending moment. At repeated loading when deep beam specimens were subjected to applied load, similar cracks that which occurred in monotonic test were observed at first cycle. At next cycles, the same cracks that were observed at the first cycle during loading phase were gradually widened and propagated diagonally along the main strut, at the last cycle, beams loaded up to failure. Finally, failure occurred by splitting

the inclined line joining the edge of steel plates at the supports and loading positions (strut of the deep beam). Plates (4) to (7) show modes of failure and the crack patterns of the tested deep beams.

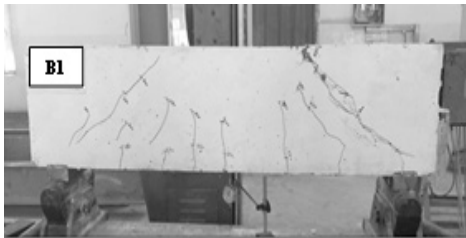


Plate 4. Crack Pattern for Beam B1 after Testing.



Plate 5. Crack Pattern for Beam B2 after Testing.

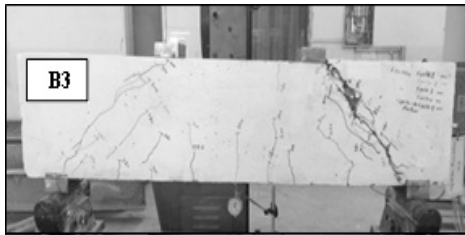


Plate 6. Crack Pattern for Beam B3 after Testing.

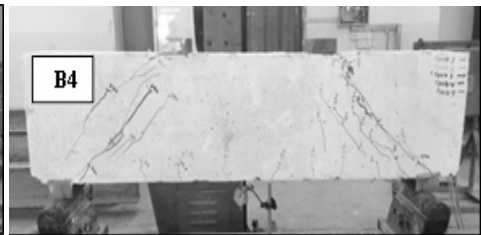


Plate 7. Crack Pattern for Beam B4 after Testing.

3.1 Effect of Loading Level per Cycle on Ultimate Load of Tested Beams

The effect of this parameter is studied for constant SF ratio and amount of web reinforcement. Hybrid beam B1 tested under monotonic loading while others beam (B2, B3 and B4) tested under repeated loading with different percentages of load level of control beam load. Results of the ultimate load are drawn in Figure (2) and listed in Table (4).

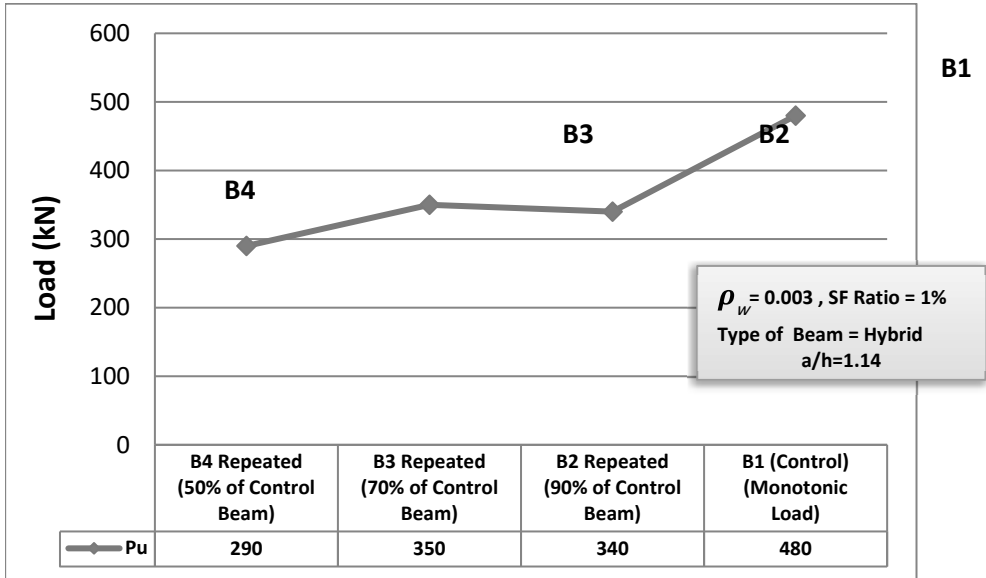


Figure 2. Effect of Loading Level per Cycle on Ultimate Loads of Beams under Repeated Loading.

Table 4. Summary of Test Results for Tested Deep Beams*.

Beam No.	Beam Type	w^{**}	Steel Fiber Ratio (SF)	Type of Loading	No. of Cycles	Ultimate Load (kN)	Modes of Failure
B1	Hybrid	0.003 (Min) ***	1	Monotonic	-	480	Diagonal Shear Failure
B2	Hybrid	0.003 (Min)	1	Repeated (90% of B1 Ultimate Load)	3	340	Diagonal Shear Failure
B3	Hybrid	0.003 (Min)	1	Repeated (70% of B1 Ultimate Load)	5	350	Diagonal Shear Failure
B4	Hybrid	0.003 (Min)	1	Repeated (50% of B1 Ultimate Load)	5	290	Diagonal Shear Failure

* All beams have the same (a/h) ratio = 1.14

** $w =$

*** Min web reinforcement ratio = 0.003 for all tested beams.

3.2 Load-Deflection Response

Figures (3) through (6), show the load mid-span deflection curves obtained for the all tested deep beam specimens which were tested under monotonic and repeated loading. The load mid-span deflection curves are initiated in a linear form with a constant slope. After initiating cracks, the load-deflection response takes a nonlinear form with variable slope where the deflection is increased at an increasing rate as the applied load is increased.

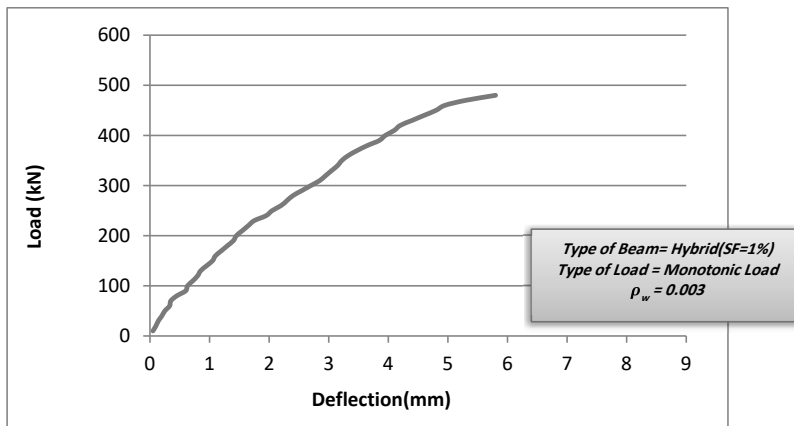


Figure 3. Load mid-span Deflection for Beam B1.

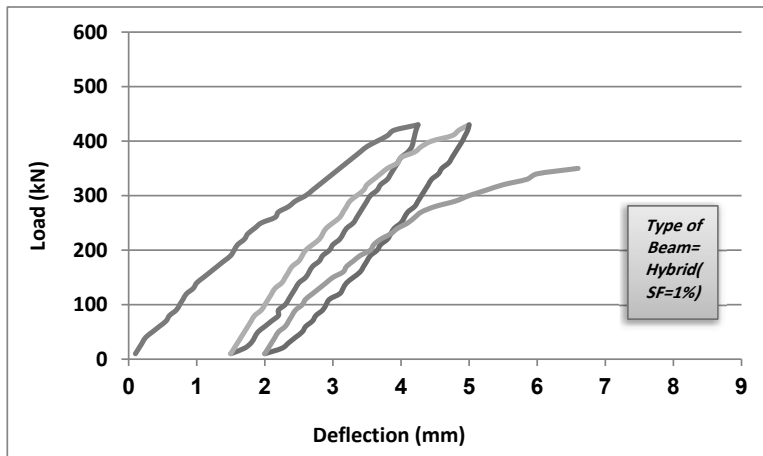


Figure 4. Load mid-span Deflection for Beam B2.

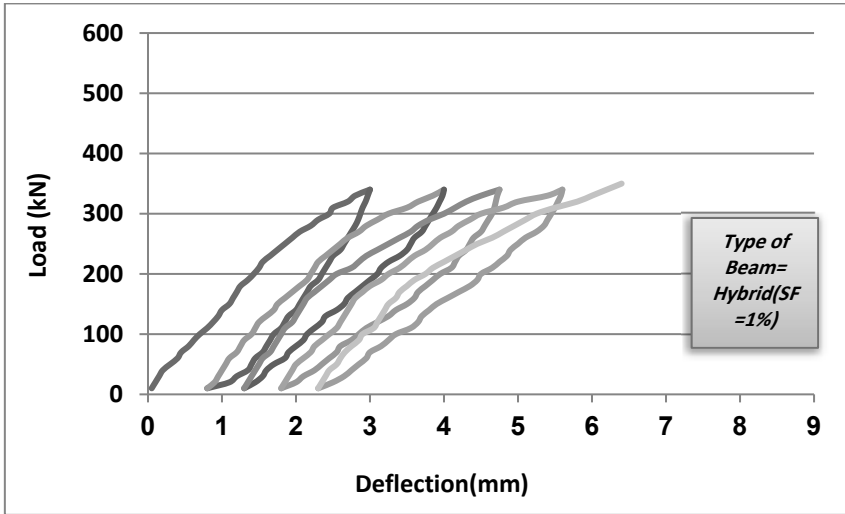


Figure 5. Load mid-span Deflection for Beam B3.

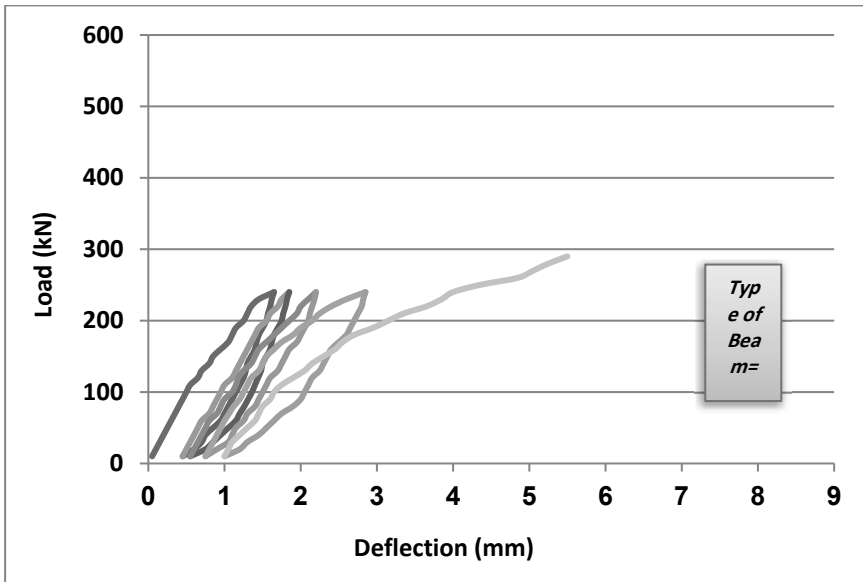


Figure 6. Load mid-span Deflection for Beam B4.



4. Theoretical Program

The Strut and Tie modeling (STM) technique is a widely accepted design approach for reinforced concrete deep beams. Where geometrical discontinuity exists in structural members, current code documents provide little direction for design. The design of these structural concrete members can be better understood by using STM (Maxwell & Breen, 2000 : 142). In this work, three methods will be used to predict ultimate load of simply supported deep beam (B1) tested under two point monotonically loading system which are:

1. Strut and tie model according to ACI 318M-11Code procedure.
2. Modified strut and tie method proposed by Zhang and Tan in March 2007.
3. Modified strut and tie model proposed by Zhang and Tan in November 2007, which takes into account size effect of reinforced concrete deep beams.

4.1 Theory of STM

STM refer to the complex flow of stresses in structural members as axial elements in a truss. Concrete struts are resisting the compressive stress and reinforcing steel ties are resisting the tensile stress. The intersection regions of struts and ties are called nodes. Struts, ties, and nodes as shown in Figure (7) are the three elements that consist the STM and they should be proportioned to resist the applied forces. The capacity of a STM is according to the lower bound theory of plasticity is always less than the structure's actual capacity provided the truss is in equilibrium and safe. Failure of a STM can be attributed

to crushing of the struts, crushing of concrete at the face of a node, yielding of the ties, or anchorage failure of the ties (Birrcher et al, 2007 : 2987).

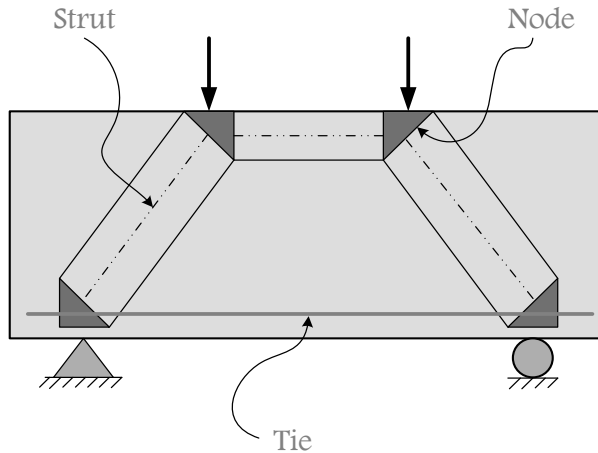


Figure 7. STM of Deep Beam.

STM in deep beams is represented by a structural truss as shown in Figure (7). Each type of the elements in a STM serves a unique purpose, but must be act in concert to describe accurately the behavior of a structure. STM consists of the following members and parts:

- a. Struts: Struts are the compression members in the STM. Struts vary in shape. Most struts in a two dimensional STM are bottle-shaped, they spread laterally along their length. The lateral spreading of a bottle-shaped strut introduces tensile stresses transverse to the strut. Transverse reinforcement should be provided in order to control the cracking along the length of the strut that which occurs as a result of the tensile stresses.
- b. Ties: Reinforcing steel bars are set at tie locations in an STM. Ties are the tension members in the STM. The reinforcement should be



distributed so that its centroid coincides with the tie location. Details such as distribution, bar spacing, and anchorage are factors that deserve the most consideration when selecting and placing the reinforcement.

- c. Nodes: Nodes form where struts and ties cross. Nodes are named according to the nature of the elements that frame into them. For example, the nodal zone where two struts and a tie intersect is referred to as a CCT node (C stands for compression and T stands for tension). Nodes are classified as CCC, CCT, CTT, or TTT.

4.2 Theoretical Results

4.2.1 Capacity of the Tested Beams Using the STM

Ultimate load for deep beam (B1) was tested under monotonic loading is calculated according to ACI 318 M-11Code. Table (5) shows the comparison between test result and the predicted value of the ultimate load. From Table (5), it can be noticed that the STM mentioned in ACI318M-11 Code underestimates the load capacity of deep beam (B1). The value (X') for the ratio of analytical/test result of ultimate load (P_{An}/P_{Exp}) is 0.68 where P_{An} refers to ultimate load obtained using analytical methods.

Table 5. Comparison between Experimental Ultimate Load and it Calculated Using STM of ACI 318M-11Code.

Beam No.	w	Beam Type	SF Ratio	Ultimate Load(2Vn) (kN)		% PAn/ PExp.
				STM ACI 318M-11 Code	Experimental Value	
B1	0.003	Hybrid Beam	1%	325.01	480	0.68



4.2.2 Modified of STM Theory

Zhang and Tan in March (2007), suggested a modified STM for calculation of shear strength of reinforced concrete deep beams based on a previous fulfillment reported by Tan and Cheng. For simply supported reinforced concrete beams subjected to symmetric two point loads, from the structural analysis it is well known that the ultimate load (P) is equal to twice the shear force at the support.

$$P = 2V_n \quad (1)$$

The expression for calculated the shear strength V_n according to Zhang and Tan, is as follows:

$$V_n = \frac{1}{\frac{4 \sin \theta_s \cos \theta_s}{A_c f_t} + \frac{\sin \theta_s}{A_{str} f'_c}} \quad (2)$$

where;

V_n : shear strength of deep beams (N).

A_c : is the beam effective cross- sectional area in mm², equals to $b_w d_c$.

d_c : effective beam depth (mm).

A_{str} : cross-sectional area of the concrete diagonal strut in mm², equal to $w_s b_w$.

w_s : effective width of the inclined strut (mm).

b_w : width of deep beam (mm).

f_τ : combined tensile strength of reinforcement and concrete (MPa).

θ_s : angle between the axis of the strut and the horizontal axis of the member.

It can be noted that the expression is the composite tensile strength included contributions from concrete and reinforcement (web and main bars), where;



$$f_t = f_{ct} + f_{st} \quad (3)$$

f_{ct} : represents the contribution of concrete tensile strength.

f_{st} : represents the contribution of steel reinforcement which consists of two parts,

f_{sw} from the web reinforcement and f_{ss} from the longitudinal reinforcement as explain in equation (4).

$$f_{st} = f_{sw} + f_{ss} \quad (4)$$

Zhang and Tan suggested that the presence of web reinforcement in the strut restricts the inclined cracks from readily increase to every ends of the strut. Equation (5) shows the tensile contribution of web reinforcement at the interface of the nodal zone.

$$f_{sw} = \frac{A_{sw} f_{yw} \sin(\theta_s + \theta_w)}{A_c / \sin \theta_s} \quad (5)$$

For concerted cases of vertical and horizontal web reinforcement, equation (5) is reduced to:

$$f_{sw} = \frac{A_{sv} f_{yv} \sin 2\theta_s}{2A_c} + \frac{A_{sh} f_{yh} \sin^2 \theta_s}{A_c} \quad (6)$$

Where:

A_{sv} : total areas of vertical web reinforcement within the shear span (mm²).

A_{sh} : total areas of horizontal web reinforcement within the shear span (mm²).

f_{yv} : tensile yield strength of vertical web reinforcement (MPa).

f_{yh} : tensile yield strength of horizontal web reinforcement (MPa).

θ_s : angle between the axis of the strut and the horizontal axis of the member.

θ_w : angle between the web reinforcement and the horizontal axis of beams at the intersection of the reinforcement and the diagonal strut.



The expression f_{ss} refers to the contribution of bottom longitudinal steel, it can be obtained according to the following equation:

$$f_{ss} = \frac{4A_s f_y \sin \theta_s}{A_c / \sin \theta_s} \tag{7}$$

Where:

A_s : total areas of bottom longitudinal main reinforcement (mm²).

f_y : tensile yield strength of main reinforcement (MPa).

Table (6) summarized the strength of the deep beam (B1) of the present investigation.

Table 6. Comparison between Experimental Ultimate Load and it Calculated Using Modified STM Theory by N. Zhang and K.H. Tan.

Beam No.	$V_n = \frac{1}{\frac{4 \sin \theta_s \cos \theta_s}{A_c f_t} + \frac{\sin \theta_s}{v A_{str} f'_c}}$	Ultimate Load(2Vn) (kN)		% PAn/PExp.
				Modified STM by Zhang and Tan	Experimental Value	
B1	0.003	Hybrid Beam	1%	609.59	480	1.27

From the above results, modified STM overestimates ultimate load as compared to test result. The X' for PAn/PExp ratio is 1.27.

4.2.3 Size Effect on the Capacity of Deep Beams Using the STM

Zhang and Tan in November (2007) suggested the following modification to equation (2) for ultimate shear strength, taking into account the size effect.

$$V_n = \frac{1}{\frac{4 \sin \theta_s \cos \theta_s}{A_c f_t} + \frac{\sin \theta_s}{v A_{str} f'_c}} \tag{8}$$

The term v refers to the efficiency factor accounts for the effect of strut geometry, and the effect of strut boundary conditions influenced by web reinforcement. The term v is expressed as follows:

$$v = \xi \times \zeta \quad (9)$$

where;

ξ : efficiency factor for the effect of strut geometry.

ζ : efficiency factor for the effect of strut boundary conditions influenced by web reinforcement. These parameters are expressed as follows:

$$\xi = 0.8 + \frac{0.4}{\sqrt{1+(l-w_s)/50}} \quad (10)$$

$$\zeta = 0.5 + \sqrt{k d_s/l_s} \leq 1.2 \quad (11)$$

Where,

l : length of strut in mm, as shown in Figure (8).

d_s : diameter of web steel bar, when web steel is not provided, d_s is taken as the minimum diameter of bottom longitudinal steel bars.

l_s : maximum spacing of web steel intercepted by the inclined strut, when web steel is not provided, l_s is equal to l .

$k = 0.5 \times \sqrt{\pi f_y/f_{ct}}$ is a material factor, when web steel is not provided, it is taken as half of the above value.

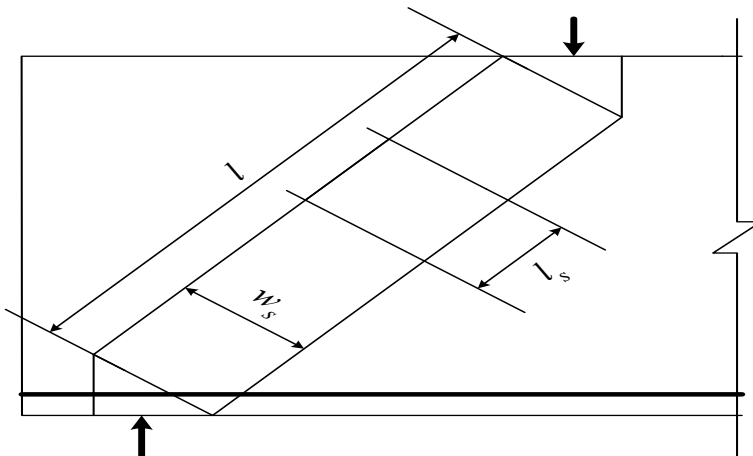


Figure 8. Strut Geometry and Strut Boundary Conditions.

Table (7) summarized the strength of deep beam (B1) of the present investigation which was tested under monotonic loading. The X' for PAn/PExp ratio is 1.28.

Table 7. Comparison between Experimental Ultimate Load and it Calculated Using Equation (8).

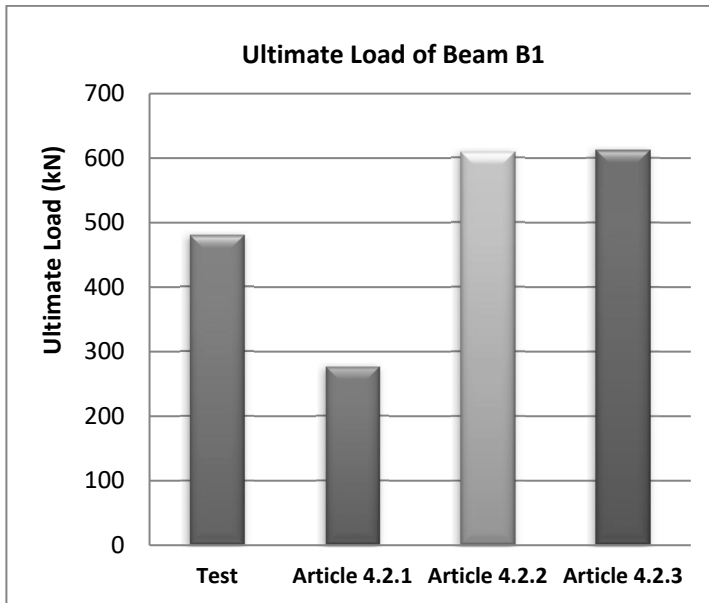
Beam No.	ρ_w	Beam Type	SF Ratio	Ultimate Load(2Vn) (kN)		% PAn/PExp.
				Modified STM Equation (8)	Experimental Value	
B1	0.003	Hybrid Beam	1%	613.21	480	1.28

5. Comparison between Ultimate Loads Obtained Using Different Analytical Methods and Experimental Values.

Through the analytical methods presented in the previous articles, it was to obtain different values of ultimate load as compared with the experimental data of the reinforced concrete simply supported deep beam (B1) that was tested under monotonic load in this work. These differences in results are shown in Table (8), Figure (9).

Table 8. Comparison between Ultimate Load Obtained Using Experimental Results and Different Analytical Methods.

Beam No.	PExp. Experimental (kN)	PAn.		
		STM ACI 318M-11 Code (kN)	Modified STM by Zhang and Tan (kN)	Modified STM Using Equation (8) (kN)
B1	480	325.01	609.59	613.21
$X'(PAn/PExp.)$		0.68	1.27	1.28



**Figure 9. Ultimate Loads Obtained of B1
Using Different Experimental and Analytical Methods.**

6. Conclusions

1. Generally, the ultimate loads of deep beams subjected to repeated loading are lower than the ultimate load of corresponding beam tested under monotonic loading. The shear strength of hybrid deep beams decreases when they subjected to repeated loading of different load levels of monotonic control beam. It was observed that the percentage decrease in ultimate load are 39.58%, 27.08 and 29.17% when hybrid deep beams are subjected to repeated load of levels 50%,70% and 90% of its control monotonic ultimate loads.



1. Strut and Tie Model presented in Appendix A of ACI 318M-11 Code results in conservative value of ultimate load as compared with the corresponding experimental ones. It can be noticed that the STM underestimates the load capacity of deep beam (B1). The value (X') for the ratio of analytical/test results ultimate loads (P_{An}/P_{Exp}) is 0.68 where P_{An} refers to ultimate loads obtained using analytical methods.
2. The modified developed by (Zhang and Tan in March 2007) gives capacities, which overestimate value as compared with STM presented in Appendix A of ACI 318M-11 Code. The X' is 1.27.
3. The modified expression developed by (Zhang and Tan in November 2007), that includes the effect of size factor, gives capacities for deep beam (B1), which overestimate value as compared with STM presented in Appendix A of ACI 318M-11 Code. The expression, also gives capacity which is convergent value for article (4.2.2) . The X' is 1.28.



7. References

- ACI Committee 318M-318RM, (2011) "Building Code Requirements for Structural Concrete and Commentary", American Concrete Institute, Farmington Hills, Michigan, 503 pp.
- S.Y. Noh, C.Y. Lee and K. M. Lee, (2009)"Deep Beam Design Using Strut-Tie Model", University at Ansan, Korea.
- B. S. Maxwell and J. E. Breen,(2000) "Experimental Evaluation of Strut-and-Tie Model Applied to Deep Beam with Opening ", ACI Structural Journal, Vol.97, No.1, pp. 142-148.
- D. Birrcher, R. Tuchscherer, M. Huizinga, O. Bayrak, S. Wood and J. Jirsa, (2008)"Strength and Serviceability Design of Reinforced Concrete Deep Beams", CTR Technical Report.
- N. Zhang, and K.H. Tan,(2007) "Direct Strut-and-Tie Model for Single Span and Continuous Deep Beams", Science Direct, Engineering Structures Journal, Vol.29, pp. 2987-3001.
- Tan, K. H., and Lu, H. K.,(1999) "Shear Behavior of Large Reinforced Concrete Deep Beams and Code Comparisons", ACI Structural Journal, Vol. 96, No. 5, pp. 836-845.
- N. Zhang, and K.H. Tan, (2007)"Size Effect in RC Deep Beams: Experimental Investigation and STM Verification", Science Direct, Engineering Structures Journal, Vol.29, pp. 3241-3254.